Integration by Substitution (cont.)

ex $\int \frac{3x}{(6x^2+4)^2}\,dx$

$u = 6x^2+4$

$\frac{du}{dx} = 12x$

$du = 12x\,dx$

$= \frac{1}{4}\int \frac{4\cdot 3x}{(6x^2+4)^2}\,dx$

$= \frac{1}{4}\int \frac{1}{u^2}\,du$

$= \frac{1}{4}\int u^{-2}\,du$

$= \frac{-1}{4}u^{-1} + C$

$\boxed{= -\frac{1}{4}(6x^2+4)^{-1} + C}$

ck: $\frac{d}{dx}\left[-\frac{1}{4}(6x^2+4)^{-1} + C\right]$

$= \frac{1}{4}(6x^2+4)^{-2}\cdot 12x + 0$

$= \frac{3x}{(6x^2+4)^2}$ ✓

ex $\quad -\int -\frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$= -\int \frac{1}{u}\,du$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln|(\cos x)^{-1}| + C$

$= \ln|\sec x| + C$

☆ $\int \tan x\,dx = \ln|\sec x| + C$

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ex

$$2\cdot\int \tfrac{1}{2}\tan^7(x/2)\sec^2(x/2)\,dx$$

options for u:

$u = x/2$

$\boxed{u = \tan(x/2)}$

$u = \sec(x/2)$

$$\frac{du}{dx} = \sec^2(x/2)\cdot\tfrac{1}{2}$$

$u = \tan x/2$

$du = \sec^2(x/2)\cdot\tfrac{1}{2}\cdot dx$

$$= 2\int u^7\,du$$

$$= 2\cdot\tfrac{1}{8}u^8 + C$$

$$\boxed{= \tfrac{1}{4}\left(\tan\tfrac{x}{2}\right)^8 + C}$$

BOUNDS!

ex $\displaystyle\int_{-3}^{3} x\sqrt{9-x^2}\,dx$

OPTION 1: $F(3)-F(-3)$

$F(x) = \int x\sqrt{9-x^2}\,dx$.

That is, do the bounds last.

OPTION 2: I'm going to u-land!!!

x-values

$-\frac{1}{2}\displaystyle\int_{-3}^{3} -2x\sqrt{9-x^2}\,dx$

x-values

$u = 9-x^2$

$du = -2x\,dx$

u-value

$= -\frac{1}{2}\displaystyle\int_{0}^{0} \sqrt{u}\,du$

u-value

when $x=-3$
$u=0$

when $x=3$
$u=0$

$=0$

ex $-\frac{1}{2}\int_0^5 -2x\sqrt{25-x^2}\,dx$

$u = 25 - x^2$

$du = -2x\,dx$

$= -\frac{1}{2}\int_{25}^{0} \sqrt{u}\,du$

If $x = 0$
$u = 25 - 0$
$= 25.$

$= \frac{1}{2}\int_0^{25} u^{1/2}\,du$

If $x = 5$
$u = 0.$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^{25}$

$= \frac{1}{3}25^{3/2} - \frac{1}{3}0^{3/2}$

* u-values for bounds and a u expression.

$= \frac{125}{3}$

$$\int_0^5 x\sqrt{25-x^2}\,dx$$

$$= -\frac{1}{2}\int_0^5 \sqrt{u}\,du$$

WRONG!

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^5$$

RIGHT

$$= -\frac{1}{3}(25-x^2)^{3/2}\Big|_0^5$$

BAD STEP!

$$= -\frac{1}{3}0^{3/2} - -\frac{1}{3}(25)^{3/2}$$

$$\int_0^3 \frac{5e^{2x}}{7+9e^{2x}}\,dx$$

$u = 7 + 9e^{2x}$

$du = 18e^{2x}dx$

$$= \frac{5}{18}\int_0^3 \frac{18e^{2x}}{7+9e^{2x}}\,dx$$

$$= \frac{5}{18}\int_{16}^{7+9e^6} \frac{1}{u}\,du$$

$$= \frac{5}{18}\ln(7+9e^6) - \frac{5}{18}\ln 16$$

$$= \frac{5}{18}\ln\left(\frac{7+9e^6}{16}\right)$$