Technique # 2 : (Integrating by Parts)

$$\int u\,dv = uv - \int v\,du$$

"Undoing the product rule!"

Recall : $\frac{d}{dx}[uv] = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$$uv = \int \left(u\frac{dv}{dx} + v\frac{du}{dx} \right) dx$$

$$uv = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx$$

$$uv = \int u\,dv + \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

An integral = another integral + stuff.

$\int u\,dv = uv - \int v\,du$

ex

$$\int \underbrace{x}_{u}\,\underbrace{\cos x\,dx}_{dv}$$

$u = x \qquad v = \sin x$

$du = dx \qquad dv = \cos x\,dx$

$$= \underset{u}{x}\cdot\underset{v}{\sin x} - \int \underset{v}{\sin x}\,\underset{du}{dx}$$

$$\boxed{= x\sin x + \cos x + C}$$

ck: $\frac{d}{dx}\left[x\sin x + \cos x + C\right]$

$$= \cancel{1\sin x} + x\cdot\cos x - \cancel{\sin x} + 0$$

$$\shortparallel$$

☺

$$\int u\,dv = uv - \int v\,du$$

ex $\displaystyle\int \underbrace{3x}_{u} \cdot \underbrace{e^{2x}\,dx}_{dv}$

$u = 3x \qquad v = \frac{1}{2}e^{2x}$

$du = 3\,dx \qquad dv = e^{2x}\,dx$

$$= 3x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} \cdot 3\,dx$$

$$= \frac{3}{2}xe^{2x} - \frac{3}{2}\int e^{2x}\,dx$$

$$= \frac{3}{2}xe^{2x} - \frac{3}{2}\cdot\frac{1}{2}e^{2x} + C$$

$$\boxed{= \frac{3}{2}xe^{2x} - \frac{3}{4}e^{2x} + C}$$

ex $\int x^2 \sin 2x \, dx$

$u = x^2$ $\quad$ $v = -\frac{1}{2}\cos 2x$

$du = 2x \, dx$ $\quad$ $dv = \sin 2x \, dx$

$$= -\frac{1}{2}x^2\cos 2x - \int -\frac{1}{2}\cos 2x \cdot 2x \, dx$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{\cancel{2}}{2}\int x\cos 2x \, dx$$

$u = x$ $\quad$ $v = \frac{1}{2}\sin 2x$

$du = dx$ $\quad$ $dv = \cos 2x \, dx$

$$= -\frac{1}{2}x^2\cos 2x + \left[\frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \, dx\right]$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

ex $\int x^4 e^x \, dx$

"Tabular method"
What's that?