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$$\int \frac{dx}{\cot 3x}$$

$$= \int \frac{1}{\cot 3x}\, dx$$

$$= \int \tan 3x\, dx$$

$$= -\frac{1}{3}\int \frac{-3\sin 3x}{\cos 3x}\, dx$$

$u = \cos 3x$

$du = -3\sin 3x\,dx$

$$= -\frac{1}{3}\int \frac{1}{u}\, du$$

$$= -\frac{1}{3}\ln|u| + C$$

$$= -\frac{1}{3}\ln|\cos 3x| + C$$

$$\int \sec x \, dx \qquad \text{hint: } \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \int \sec x \cdot \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$$= \ln |\sec x + \tan x| + C$$

since we had $\int \frac{u'}{u}$

$$\frac{3}{4}\int \underline{\frac{4}{3}x^{1/3}}\cos\left(x^{4/3}-8\right)\underline{dx}$$

$$=\frac{3}{4}\int \cos u \, du$$

$$u = x^{4/3} - 8$$

$$\boxed{du = \frac{4}{3}x^{1/3}\,dx}$$

$$=\frac{3}{4}\sin u + C$$

$$=\frac{3}{4}\sin\left(x^{4/3}-8\right)+C$$

Technique #2 Integration by Parts.

$$\int u\,dv = uv - \int v\,du$$

$\star$ Undoing the Product Rule $\star$

Recall:

$$\frac{d}{dx}[uv] = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$$

$$uv = \int\left(u\frac{dv}{dx} + v\frac{du}{dx}\right)dx$$

$$uv = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx$$

$$uv = \underbrace{\int u\,dv} + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Idea: An integral can be written as another integral + stuff.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

ex $\int \underbrace{x}_{u} \underbrace{\cos x\,dx}_{dv}$

$u = x \qquad v = \sin x$

$du = dx \qquad dv = \cos x\,dx$

$= \underbrace{x}_{u} \cdot \underbrace{\sin x}_{v} - \int \underbrace{\sin x}_{v}\,\underbrace{dx}_{du}$

$$\boxed{= x\sin x + \cos x + C}$$

ck: $\frac{d}{dx}\left[x\sin x + \cos x + C\right]$

$= 1 \cdot \cancel{\sin x} + x \cdot \cos x - \cancel{\sin x} + 0$

$$\int u\,dv = uv - \int v\,du$$

ex

$$\int \underbrace{3x}_{u}\ \underbrace{e^{4x}\,dx}_{dv}$$

$u = 3x$ $\qquad$ $v = \frac{1}{4}e^{4x}$

$du = 3\,dx$ $\qquad$ $dv = e^{4x}\,dx$

$$= 3x \cdot \frac{1}{4}e^{4x} - \int \frac{1}{4}e^{4x} \cdot 3\,dx$$

$$= \frac{3}{4}xe^{4x} - \frac{3}{4}\int e^{4x}\,dx$$

$$= \frac{3}{4}xe^{4x} - \frac{3}{4}\cdot\frac{1}{4}e^{4x} + C$$

$$\boxed{= \frac{3}{4}xe^{4x} - \frac{3}{16}e^{4x} + C}$$

ex $\int x^2 e^x dx$

$u = x^2$ $v = e^x$

$du = 2x\,dx$ $dv = e^x dx$

$= x^2 e^x - \int e^x 2x\,dx$

$= x^2 e^x - 2\int x e^x\,dx$

$u = x$ $v = e^x$

$du = dx$ $dv = e^x dx$

$= x^2 e^x - 2\left(x \cdot e^x - \int e^x dx\right)$

$$\boxed{= x^2 e^x - 2xe^x - 2e^x + C}$$

<u>ex</u> $\int x^5 \cos x \, dx$

"Tabular Method".

idea: $uv - \int v\,du$

$uv - (uv_1 - \int v_1\,du_1)$

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