$$\frac{d}{dx}\int_0^{x^2} t+e^t\,dt$$

$$2x\left(x^2+e^{x^2}\right)$$

$$\frac{d}{dx}\int_0^{x} t+e^t\,dt = x+e^x$$

$y = h(x)$ where $h(x) = \int_0^x t+e^t\,dt$

$$\frac{dy}{dx} = x+e^x$$

Suppose $y = h(x^2)$ Let $u = x^2$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$h(x^2) = \boxed{\int_0^{x^2} t+e^t\,dt}$$

Do not look at this

$y = f(x)$ then $\frac{d}{dx}(y) = \frac{dy}{dx}$

$y = f(u)$ then $\frac{d}{dx}(y) = \frac{dy}{du}\cdot\frac{du}{dx}$

$$\frac{d}{dx}\left[\int_0^{x^2} t+e^t\,dt\right] = \frac{d}{du}\left[\int_0^{u} t+e^t\,dt\right]\cdot\frac{du}{dx}$$

$$= (u+e^u)\cdot 2x$$

$$= \left(x^2+e^{x^2}\right)2x$$

$$h(g(x)) = \int_0^{g(x)} f(t)\,dt$$

Suppose c is in $[a, b]$

Then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\frac{d}{dx}\int_x^{x^2} \cos t \, dt$$

$$\frac{d}{dx}\left[\int_x^{a} \cos t \, dt + \int_a^{x^2} \cos t \, dt\right]$$

$$\frac{d}{dx}\left[\int_a^{x^2} \cos t \, dt - \int_a^{x} \cos t \, dt\right]$$

$$2x\cos x^2 - \cos x$$

MVT for Integrals

Suppose f is continuous on $[a,b]$

There is some c in $[a,b]$

such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$