$$\int \overbrace{3x^2 \underbrace{\sqrt{x^3+1}}_{\sqrt{u}}\ dx}^{du}$$

$u = x^3 + 1$

$du = \underbrace{3x^2\,dx}$

$\int \sqrt{u}\ \underline{3x^2\,dx}$

(1) Do you see a function in a function?

(2) Do you see its derivative "hanging around"?

$\hookrightarrow$ within a constant factor

$$\int \sqrt{u}\ du$$

$$\int u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} + C$$

$$= \frac{2\sqrt{x^3+1}^{\,3}}{3} + C$$

$$\int 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3+1$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2\,dx$$

$$\underline{\enclose{circle}{dx} = \frac{1}{3x^2}\,du}$$

$$\int \sqrt{u}\cdot 3x^2 \cdot \frac{1}{3x^2}\,du$$

$$\int \sqrt{u}\,du$$

$$\int x^n \, dx \qquad \int x^{\frac{1}{2}} \, dx$$

$$\frac{x^{n+1}}{n+1} + C \qquad \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\frac{2}{3} \cdot x^{\frac{3}{2}}$$

$$\frac{1}{3}\int 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3+1$

$du = 3x^2\,dx$

$$\frac{1}{3}\int \sqrt{u}\,du$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3 + 1$

$du = 3x^2\,dx$

$u = (-1)^3 + 1$

$= 0$

$u = 1^3 + 1$

$= 2$

$$\int_{0}^{2} \sqrt{u}\,du$$

$$\frac{2}{3}u^{\frac{3}{2}}\Big|_0^2 = \frac{2}{3}(2)^{\frac{3}{2}} - 0$$

$$= \frac{2\sqrt{8}}{3}$$

$\sqrt{8} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

$$= \frac{4\sqrt{2}}{3}$$

$$\int f(x)\,dx = F(x) + C$$

where $F'(x) = f(x)$

$$\int_a^b f(x)\,dx$$

$$\int \quad dx$$

$$\frac{dy}{dx} = 2x\,(1+y^2)\,e^{x^2}$$

$$\int \frac{1}{1+y^2}\frac{dy}{dx}\,dx = \int 2x e^{x^2}\,dx$$

$$\int \frac{1}{1+y^2}\,dy = \int 2x e^{x^2}\,dx \qquad \text{Let } u = x^2$$

$$du = 2x\,dx$$

$$\tan^{-1} y + c_1 = \int e^u\,du$$

$$\tan^{-1} y + c_1 = e^u + c_2$$

$$\tan^{-1} y + c_1 = e^{x^2} + c_2$$

$$\tan^{-1} y = e^{x^2} + C$$

$$y = \tan\left(e^{x^2} + C\right)$$